Attitude Determination of GPS Satellite Vehicles

A Thesis

Presented to

the Faculty of the Department of Engineering Technology

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Master of Science

in Engineering Technology

By

Mrinalini Arcot

August, 2014

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Abstract

There is an increasing demand for navigation systems that has led to rapid development of Global Positioning System (GPS) across industries. Apart from position and speed, precise attitude measurements are needed for many GPS applications. This thesis presents techniques for attitude determination of satellite vehicles in both real-time and stand-alone positioning applications. The GPS system used is a differential GPS system that estimates the body frame baselines using at least four receivers. The attitude information is obtained using these baselines and projecting them onto a local level frame. Integer ambiguity is a major constraint in attitude determination. Least Squares Ambiguity Deco-relation method is implemented to fix the ambiguities prior to baseline estimation. Estimation techniques such as Least Squares and Kalman Filter are implemented for deriving baseline components. Finally, this system will compute body frame coordinates and attitude components in reference to the desired coordinate frames.

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Chapter 1

Introduction

1.1 Introduction to Attitude

The attitude is orientation of a rigid body in space relative to a fixed reference frame. For most of the navigation and guidance purposes, the reference frames are North-East-Down (NED) and East-North-Up (ENU) coordinate frames. The coordinate frame is initially aligned to the reference frame. Attitude consists of parameters that describe rotation sequence. This rotation sequence transforms the coordinate frame to another frame that is parallel to a frame fixed to a vehicle and rotates with it. This vehicle carried frame is called the body frame. The attitude is described by parameters called *attitude coordinates,* and consists of at least three coordinates. There are several rotating sequences. One such rotation sequence is established by body-axes rotation. Rotating three times about the axes of the body's fixed reference frame establishes Euler's angles. As show in figure 1, another is orientation of the body in three dimensions about the vehicle's center of mass, establishing raw, pitch and yaw defined as below [26]:

Yaw is the angular measurement of the horizontal from with respect to the local north.

Pitch is the angular measurement of the forward direction of the body frame with its horizontal projection.

Roll is the angular measurement of the perpendicular vector to the forward direction of the body frame with its horizontal projection.

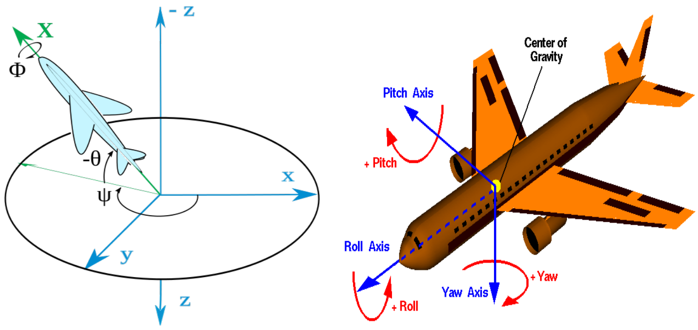

Figure 1.1. Rotation sequences and angles [1][2]

1.2 Global Positioning System

The Global Positioning System (GPS) is a satellite navigation system that provides position, navigation and time information in all weather conditions, anywhere on or near the Earth. It is robust, light and power efficient. It is maintained by the United States government and is accessible to anyone with a GPS receiver[3].

GPS has three segments namely space, user and control segments.

1. The **space** segment now consists of 24 artificial satellites, uniformly distributed in 6 orbits. They are located at an altitude of approximately 22,200 km (13,670 miles). Each satellite rotated around the Earth twice a day.
2. The **control** segment consists of stations with facilities to keep track of health of satellites and also monitor their transmissions.
3. The **user** segment consists of GPS receivers for day-to-day use to common man.

GPS was an initiative by Department of Defense, primarily for military purposes. Even though it is estimated that there are ten times as many civilian receivers as military ones the system still has considerable military significance.

1.3 Global Positioning System receivers

GPS receivers work under any weather conditions. They calculate the position by timing the signals received from SV's. Each SV transmits radio waves continuously that contain information including [4]:

- The time when message was transmitted
- Time position of satellite when message was transmitted

The receivers use these messages to compute parameters such as the transit time of each message signal and distance to each SV. Generally, receivers update their estimated location once per second.

1.4 Attitude determination using GPS

The Global Positioning System is used in various applications in space. It is used in determining the attitude, position and speed of satellite vehicles. GPS attitude determination technology uses GPS antennas and receivers to derive attitude parameters from the corresponding body to a local level frame. If the initial integer ambiguity set is solved accurately, GPS can achieve centimeter level precision. This facilitates the use of GPS sensors for attitude determination into an effective technology for many navigation missions.

Ambiguity resolution is an important step in attitude determination and is usually the first step followed by attitude estimation. If multiple closely spaced GPS antennas are mounted properly on a platform and differences of GPS signals measurements are collected simultaneously, the baseline vectors between antennas can be determined and thus, platform orientation defined by these vectors can be calculated. The closely spaced antenna configuration helps in determining the inter-antenna distance in GPS attitude determination systems and can be used as a constraint in ambiguity resolution. Thus, the prerequisite for attitude determination using GPS is to calculate baselines between antennas to millimeter level of accuracy.

The attitude determination is based on light-of-sight GPS interferometric observations to precisely estimate inter-antenna vectors in a specific navigation frame. Initially, the ambiguities for the antenna baselines are estimated in the local level frame the rotation of the antenna vectors from the body frame to the navigation frame is expressed as a rotation matrix parameterized by three Euler attitude angles. The attitude parameters can be derived from the rotation matrix using estimation techniques such as Least Squares or Kalman Filter estimation based on the known antenna coordinates both in the body and local level frames. This approach is called Baseline estimation method.

Another approach is to estimate the attitude parameters directly from the GPS observations. In this method, each single difference or double difference observable forms an independent observation equation. Compared with the baseline estimation method, this method improves redundancy and reliability of the system. However, the direct method relies on the rigidity of antenna arrays and the computation load of this method is much higher than that of the baseline estimation method [5].

In the past, GPS was used as a tool validation of navigation solutions, not for determination. Attitude determination using GPS was initially proposed by Spinney in 1976. In early 80's, Greenspan el at and others helped clarify the advantages of using GPS carrier phase. Concurrently, Brown, Bowles and Thorvaldsen proposed that carrier phase be applied to attitude determination [16]. Its use in attitude determination has gained attention in the past 10 years owing to its advantages such as accuracy and cost.

1.5 Thesis contributions

This thesis focuses on implementation of Least Squares Ambiguity Decorrelation method (LAMBDA) for determining attitude of GPS satellite vehicles. The main aim of this research work is to implement different methods of attitude determination of GPS satellite vehicles under one cluster. The contributions to this aim are as follows:

- Implementation of algorithms for integer ambiguity resolution using LAMBDA.
- Demonstration of algorithms for baseline estimation using least squares estimation and extended Kalman filter estimation.
- Determination of attitude components using the estimates obtained from the above stated methods.

1.6 Thesis organization

The remaining sections of this thesis will present the details of attitude determination with GPS.

In Chapter 2, overview of GPS including different types of measurements is reviewed. Basic mathematical models are discussed

In Chapter 3, the performance of attitude determination using GPS in terms of principal errors and noise in measurements is discussed. The effect of the geometric distribution of satellites on attitude estimation is investigated. Different differencing techniques are discussed.

In chapter 4, different integer ambiguity resolution methods and baseline component estimation methods are discussed.

In chapter 5, results from various baseline estimation methods and attitude determination are discussed.

Chapter 2

GPS overview

2.1 GPS Overview

The GPS is a satellite based navigation system consisting of space vehicles (24-32 SVs). Each SV orbit the earth in 12 hours. The multiple space vehicles are uniformly distributed in six high earth orbits such that there are four SV's per orbit. This ensures that at least eight satellites can be seen at any time from almost anywhere on earth. These earth orbits are of 22,000 km (around 13,670 miles) at almost 55-degree inclination. Figure 2.1 shows an example of arrangement of the GPS SVs in earth orbits. At any instant, location of each SV is ±1.7m. Each SV broadcasts, "radio waves" towards earth that contain information regarding its position and time. This data is collected by GPS receivers, which can detect and decode the information. By combining data from different SV's, that is received simultaneously, a GPS receiver can calculate its position on earth (in terms of longitude, latitude and height) with millimeter accuracy [4].

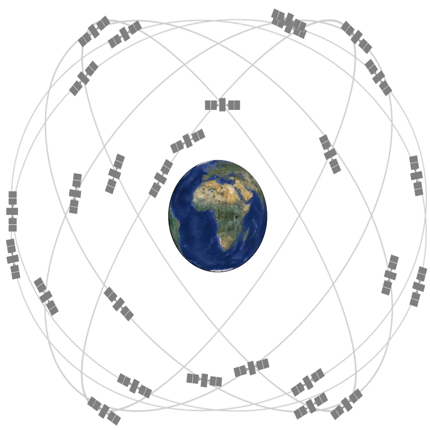

Figure 2.1: Arrangement of SV's around Earth [28]

The GPS position on orbit is in terms of number of reference frames. GPS satellites are powered by solar energy. They have backup batteries readily available to keep them running in the event of a solar eclipse or when there's no solar power. There are Small rocket boosters on each satellite keep them flying in the correct path. Figure 2.2 shows a polar plot depicting the number of visible satellites and the period of time when they are visible during 24 consecutive hours, when viewed from a given location. Figure 2.3 shows the time period during which the satellites are visible from the data used in this research.

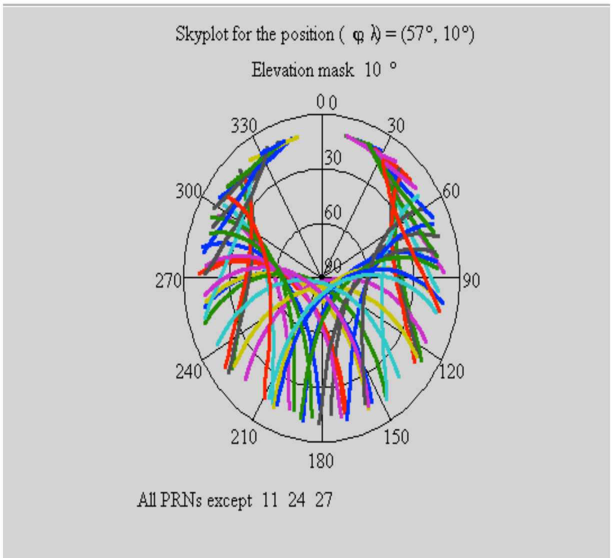

Figure 2.2: Arrangement of SV's around Earth from a particular location [9]

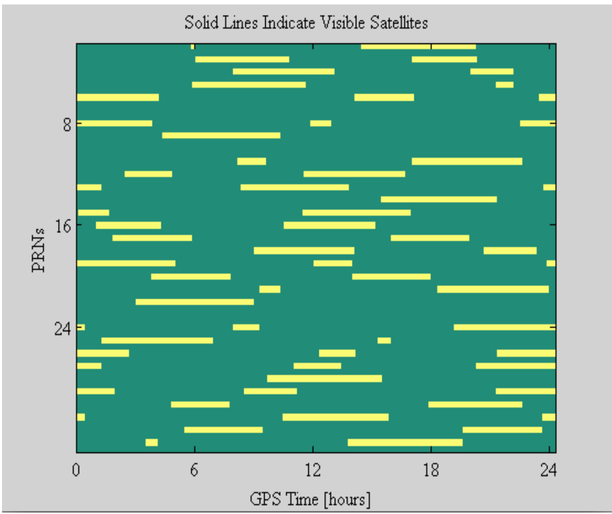

Figure 2.3: Time periods during which the satellites are visible [9]

2.2 GPS signal information

GPS signals consists of two components [6]:

- Ranging codes
- Navigation message

Ranging Codes

Ranging codes are used to measure the distance between satellite and antenna. There are two types of ranging codes:

- *Coarse/Acquisition* (C/A) code, which is available freely for commercial uses.
- *Precision* (P) code, which is restricted to military applications.

The C/A code contains a 1,023 bit deterministic sequence called pseudo random noise signal (PRN code), because they look like random noise signals, but in reality mathematical algorithms generate them. They repeat every millisecond when transmitted at 1.023 megabits per second (Mbit/s). These 1.023M Hz. These sequences only correlate when they are aligned. Each SV transmits a unique PRN code, which enables the GPS receivers to identify which satellite is transmitting a particular code. Each PRN code does not correlate well with any other satellite's PRN code. In other words, the PRN codes are highly orthogonal to one another. This is known as CDMA (Code Division Multiple Access) spread-spectrum technique, which allows the receiver to recognize multiple satellites on the same frequency. There are 1025 different Gold codes of length 1023 bits, but only 32 Gold codes with the best correlation properties are used in practice [29].

The P code is also a PRN code, which repeats once every seven days and modulated both L1 and L2 carriers at 10*MHz* rate. This code is usually encrypted due to its use in military applications. Each satellite's P-code PRN code is 6.1871×10^{12} bits long (6,187,100,000,000 bits, ~720.213 gigabytes) and only repeats once a week (it is transmitted at 10.23 Mbit/s). The extreme length of the P-code increases its correlation gain and eliminates any range ambiguity within the Solar System. However, the code is so long and complex it was believed that a receiver could not directly acquire and synchronize with this signal alone. It was expected that the receiver would first lock onto the relatively simple C/A code and then, after obtaining the current time and approximate position, synchronize with the P-code.

While the C/A PRNs are unique for each satellite, the P-code PRN is actually a small segment of a master P-code approximately 2.35×10^{14} bits in length (26.716 terabytes) and each satellite repeatedly transmits its assigned segment of the master code.

To prevent unauthorized users from using or potentially interfering with the military signal through a process called spoofing, it was decided to encrypt the P-code. To that end the P-code was modulated with the W-code, a special encryption sequence, to generate the Y-code. The Y-code is what the satellites have been transmitting since the anti-spoofing module was set to the "on" state. The encrypted signal is referred to as the P (Y) code [29].

Navigation Message

In addition to the PRN ranging codes, a receiver needs to know detailed information about each satellite's position and the network. This information is modulated on top of the two ranging codes at 50 bits/s. Since the C/A code repeats after every millisecond, 20 C/A codes will be comprised within a period of 20 millisecond i.e. before the navigation message bit changes. The navigation message is made up of three major components. The first part contains the GPS date and time, plus the satellite's status and an indication of its health. The second part contains orbital information called ephemeris data and allows the receiver to calculate the position of the satellite. The third part, called the almanac, contains information and status concerning all the satellites; their locations and PRN numbers.

2.3 Pseudorange measurements

As shown in figure 2.4, pseudo distance between a GPS satellite vehicle and a GPS receiver is known as pseudorange measurement. A receiver calculates the transmission time of the signal that is emitted form multiple SV's (4 or more) and is used to determine position and reception time. Pseudorange measurements are also called "Code phase measurements".

The pseudorange measurement of each satellite is obtained by multiplying the speed of light with the transmission time (flight time) of each SV [7].

$$\hat{\rho} = c \cdot \Delta t \qquad (2.1)$$

Where,

- $\hat{\rho}$ is the pseudorange measurement
- c is the speed of light
- Δt is the transmission time or flight time of the signal.

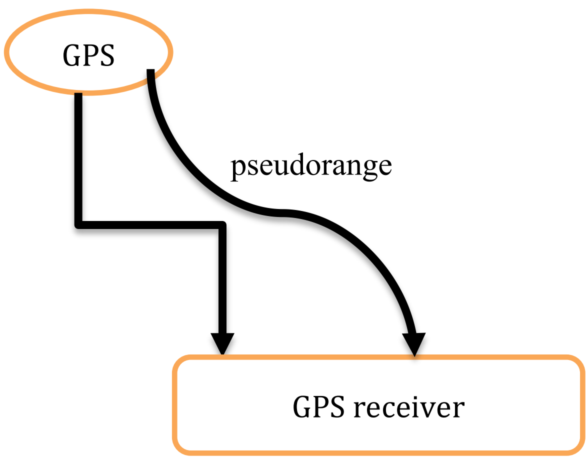

Figure 2.4: Pseudorange measurement between GPS receiver and SV

Typically, pseudorange measurement is be calculated by decoding the pseudorandom noise signal (PRN) transmitted by SV and comparing it to an identical signal locally generated by the receiver. As seen in figure 2.5, the GPS receiver determines the transmission time, Δt, by correlating the received code from the GPS SV with a replica of it that is locally generated in the receiver. This replica is moved in time (Δt) until the maximum correlation is obtained.

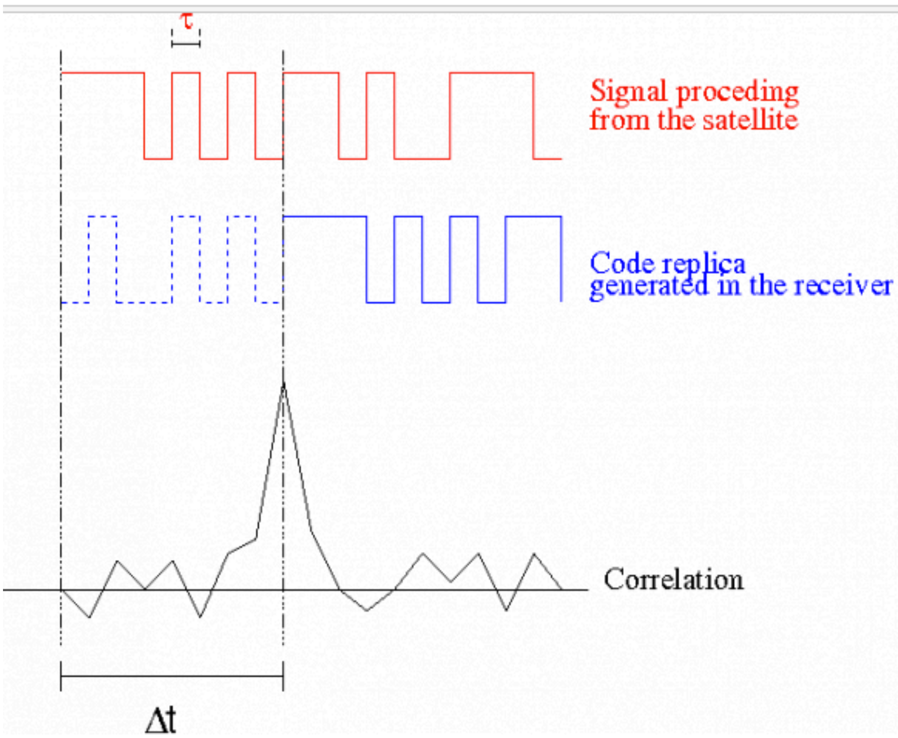

Figure 2.5: Comparison of SV signal and local signal at the GPS receiver[7]

The pseudorange measurement need not match with the actual geometric signal due to various performance factors like "synchronism errors" between the receiver clock and the satellite clock.

Let t_s be the time measured at the SV and t_r is the time measure at the receiver. The pseudorange measurement is expressed as [8]:

$$\rho(t)=c.\ [t_r(T_2)-\ t_s(T_1)] \tag{2.2}$$

Where,

- $\rho(t)$ is the pseudorange measurement
- $t_r(T_2)$ is the flight time when measure in the time scale at receiver
- $t_s(T_1)$ is the flight time when measure in the rime scale at satellite vehicle.

The equation (2.2) is valid only under ideal conditions. Hence, factors like noise, time delays, instrumental delay, geometric range etc. must also be considered while calculating pseudorange measurement. Hence, the pseudorange measurement can be rewritten as:

$$\rho(t)= \rho+c.\ [dt_r-dt_s]+errors \tag{2.3}$$

Where,

- ρ is the geometric range between GPS receiver and SV
- Errors include tropospheric delays, instrumental delays, noise errors etc.

Exact receiver position at a given time is not computed, but estimated using least squares estimation. This procedure is iterated through many epochs, to get various changing positions with time. Each position is a result of an iterative least square. Figure 2.6 shows variation of receiver positions with variation in epochs (in seconds) as calculated using pseudorange measurements.

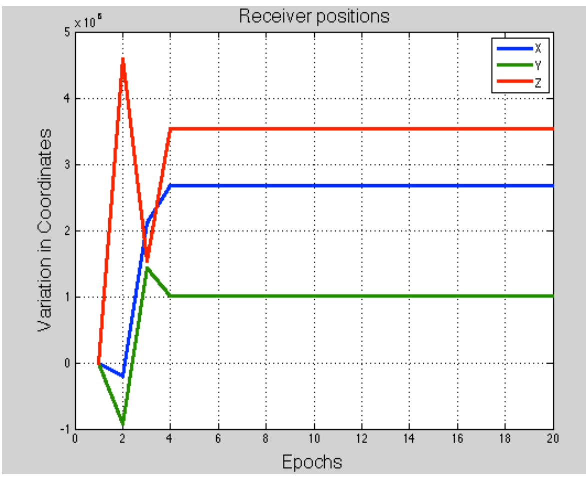

Figure 2.6: Various receiver positions measured using pseudoranges [9]

2.4 Carrier phase measurement

The carrier phase measurement is another common term to determine the position of SV. It is a measure of the range between a GPS SV and GPS receiver, expressed in units of cycles of the carrier frequency. The GPS SV's transmit radio frequency signals in L band. These signals consist of a radio frequency carrier modulated by a PRN code. When these radio frequency signals are modulated by PRN code, this resulting signal will no longer be pure sinusoidal signals. They will be a linear combination of sinusoidal signals. The phase of the signal is the offset angle of the sinusoidal carrier wave from the SV. In other words, it is the phase of the pure sinusoidal signal if PRN code and other modulations are removed. Generally, the whole number of cycles between SV and receiver is not measureable. While taking this measurement, the numbering scale returns to zero, approximately, every 20 centimeters for L1 wavelength. This allows high precision measurement (in terms of millimeters), but leaves an ambiguity in the number of whole number of carrier cycles.

The SV and the receiver are said be locked when the signal from SV reaches the receiver. At the first instant, receiver measure only a fractional phase of the carrier signal. During the next cycles, receiver keeps on measuring the fractional phase and keeps a count of number of full wavelengths that have passed. Hence, on the k^{th} instant, there will be a measured fractional phase plus measured full cycles since first instant. However, the initial number of cycles between the SV and receiver is not measure. This is known as the initial cycle ambiguity. If the SV and the receiver remain locked, then, this initial cycle ambiguity remains constant. This ambiguity plus the measured fractional cycle at a particular instant gives the total number of cycles between the SV and receiver. This measurement gives the carrier phase measurement. Carrier phase measurement is often known as Accumulated Doppler range. Under ideal conditions, multiplying carrier phase measurement with the wavelength of the carrier gives the range. To determine the range, the required terms are:

- Received radio frequency phase
- The receiver time (t_{rx})
- Initial satellite phase offset ($\emptyset^{s}(o)$)
- Integer number of cycles between SV and receiver.

The phase term is different for each satellite, and therefore, could cause significant problems in processing carrier phase measurements. However, by choosing the initial value of phase, the bias term can be constant for all the satellites. Carrier phase processing is typically implemented using double difference processing. By using this method, the clock bias is removed, thus, allows estimating the ambiguities independent of clock bias. At times, there will be a discontinuity or jump in the carrier phase measurements due to temporary signal loss. This is called cycle slip. Figure 2.7 shows the ambiguities and cycles slips that occur during the motion of GPS SV. This can occur due to receiver errors, ionosphere and tropospheric delays, obstruction in signal path etc. Cycle slips may last for minutes and may affect more than one SV signal. The size of slip may range from one cycle to million cycles. Cycle slips must be identified and corrected to avoid any errors in position computation.

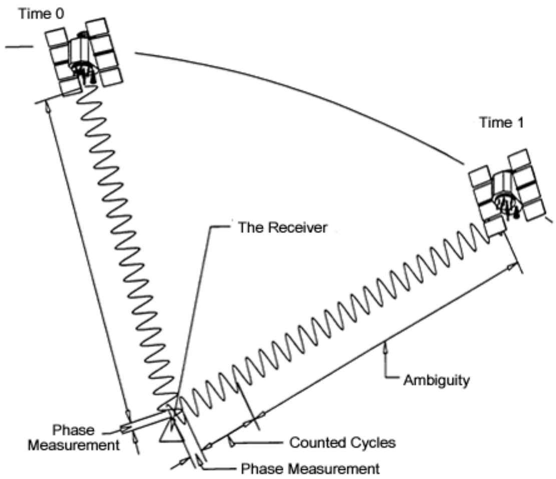

Figure 2.7: Generation of carrier phase measurement

The carrier phase receivers are accurate than the C/A code receivers. However, they require more post-processing and stricter data collection requirements. The carrier phase receivers require a clear view to the gps SVs in order to maintain constant lock with a minimum of four satellites where as the C/A receivers do not need to maintain constant lock to calculate the position parameters. This makes a C/A code receiver important in gathering data under adverse conditions. The accuracy increases by 5 meter (approximately) with carrier phase data. Once, the data is accurately processed, carrier phase measurement can be calculated. To ensure that the lock is never lost in carrier phase receivers, carrier time is observed periodically

2.5 Mathematical models for carrier phase measurement and pseudorange measurement

Time systems

There are three different time tracking systems as follows:

- True time, t. This is also known as GPS time
- Receiver clock time, t_R
- Satellite clock time, t^s

All these time systems affect the GPS measurements. True GPS time, t, is used to measure the propagation delay. The receiver clock is used to generate a carrier replica signal at the GPS receiver. The satellite clock is used to generate the original carrier signal. The true GPS time has a relationship with both receiver and satellite clock times. They are given by:

$$t = t_R - \delta t_R \tag{2.3}$$

Where, δt_R is receiver clock error.

$$t^s = t + a_{f0} + a_{f1}(t - t_{oc}) + \delta t_{rel} \tag{2.4}$$

Where, the parameters a_{f0} and a_{f1} and the epoch time t_{oc} are part of the GPS satellite navigation data message, and δt_{rel} is a relativistic correction that depends on the eccentricity, semi major axis, and eccentric anomaly of the GPS satellite's orbit.

Mathematical models for carrier phase measurement [10]

The carrier phase measurement is the difference between received carrier phase from SV and a replica generated at the receiver. Consider the model shown in figure 2.8. Let i and j be two GPS SV and A and B be two receivers at base station.

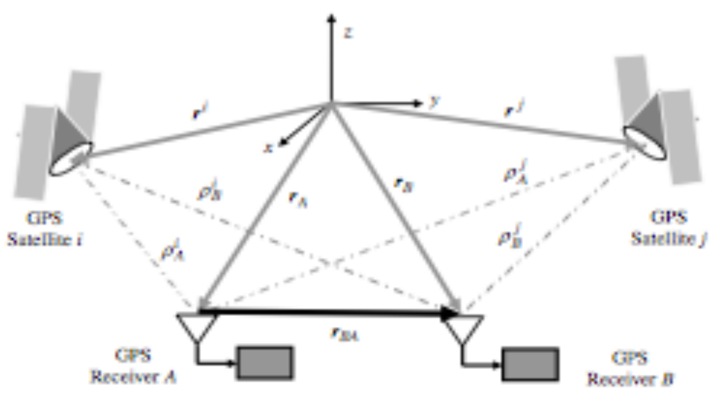

Figure 2.8: Geometry of satellites for carrier phase measurements [10]

Then, consider the following terms:

ρ_A^i , ρ_B^i, ρ^j_A, ρ_B^j are the range measurement from the receivers. Each of these measurements contains bias known as ambiguity.

- γ_A, γ_B are the absolute user antenna positions.
- γ^i, γ^j are the GPS satellite absolute positions.

Then, the carrier phase observable is given by:

$$\phi_1^A(t_{RA}) = \gamma_{1A}^j(\ (t_{RA}) - \psi_{1A}^j(t_{RA} - \delta t_{RA}) \qquad (2.5)$$

Where,

- t_{RA} is the clock time of GPS receiver A
- δt_{RA} is the error in GPS receiver
- $\psi_{1A}{}^j$ is the received phase from SV

Mathematical models for pseudorange measurement

The pseudorange model for receiver A at time t_k is given by:

$$P_{1A}{}^j = \rho^j_A + c\ .\ (\delta t_{RA} - \delta t_{RA}{}^j) \qquad (2.6)$$

$$P_{2A}{}^j = \rho^j_A + c\ .\ (\delta t_{RA} - \delta t_{RA}{}^j) \qquad (2.7)$$

Where, $P_{1A}{}^j$ and $P_{2A}{}^j$ are L1 and L2 measure pseudoranges from SV j.

The same equation holds good for receiver B.

Chapter 3

Performance factors of attitude determination and differencing techniques

3.1 Common errors in GPS measurements

There are different varieties of factors that affect the performance of GPS attitude determination [11]. There are two different kinds of errors:

- Measurement errors
- Systematic errors

Measurement Errors

The measurement errors that affect the performance of attitude determination are multipath and receiver noise errors. These errors are also known as stochastic errors.

- **Multipath Errors**

As seen in figure 3.1, when a GPS SV broadcasts a signal towards a GPS receiver, there are a lot of signal reflections and diffractions in its path. These reflections and diffractions are due to obstacles in the path. These errors are more prominent in pseudorange measurements than in carrier phase measurements. The multipath error depends on the direction of the arrival of pseudorange signal and the attitude of the GPS SV. These kinds of errors can play a significant role by limiting the accuracy of the attitude determination [5]. With the existence of multipath, the actual incoming signal at the receiver part is the composite of a direct signal and more than one reflected signals. Pseudorange multipath errors can cause a range as large as

150m, but is in the order of several meters under normal conditions. In recent years, research has been carried out successfully in developing technologies to calibrate the pseudorange multipath errors. Such technologies include TrEC (Phelts and Enge 2000), MEDLL (Townsend 1995) etc. Typical RMS levels of a carrier phase multi path error are 0.005 m/λ cycles, where λ is the wavelength.

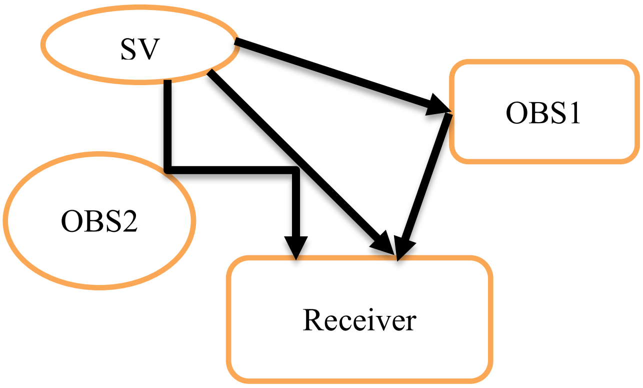

Figure 3.1: Typical cause of multi path errors

- **Receiver noise errors**

Receiver noise errors are known are carrier phase receiver noise. It consists of receiver thermal noise, induced dynamic stress etc [12]. These noises represent the measurement errors in the phase tracking loops of a GPS receiver. They are a result of random noise signals received by the antenna and generated in the receiver's radio frequency front end. Most of the errors are caused by Gaussian white noise in the receiver if the sampling frequency is less than the bandwidth of the receivers phase locked loop. The receiver noise is a function of noise is a function of received carrier to noise ratio (C/No) and the phase locked loop bandwidth BPLL [10].

The standard deviation of the receiver noise is given as:

$$\sigma_{n\phi A}^{j} = \frac{1}{2\pi}\sqrt{\frac{B_{PLL}}{2C/N_o}} \tag{3.1}$$

The receiver noise is inversely proportional to the carrier to noise ration i.e. higher the carrier to noise ratio, smaller is the receiver noise error. Note that higher carrier to noise ratio reduces thermal noise error. Phase locked loops in carrier phase are affected largely by dynamic stress. This leads to a large change in loop bandwidth in phase locked loop. The receiver noise error in a stationary GPS receiver is low since a very narrow carrier loop bandwidth is used in the phase locked loop. With increase in carrier loop bandwidth, the receiver noise error automatically increases. The carrier phase noise level is less than 1mm [13].

- **Antenna phase center errors**

Antenna phase center variations are caused by both PCO (phase center offset) and the PCV (phase center variations). These produce errors that are dependent on the direction of the arriving signal at the GPS receiver [30]. These errors also depend on the attitude of the GPS SV. These variations are translated into distance and are added as errors in the carrier phase observable. These errors lead to a perturbation of 0.01m/λcycles. These errors have same geometric dependencies as that of multipath errors.

Systematic Errors

Factors such as geometry of GPS SV, inter- antenna distance, numbers of satellites etc. also affect the carrier phase measurements. Such errors are known as systematic errors or operation errors.

- **Geometry of GPS satellite vehicles**

The number of GPS SV's in an orbit and their spacing arrangements affects the attitude estimation solution. Distance between each GPS SV and receiver is treated as an observation while estimating the attitude. Wider the spacing between GPS SV's, greater will be the accuracy of attitude estimation. Additional measurements increase the measurement redundancy but addition of good observations improves the accuracy of solution. The positioning accuracy between the receivers is dependent on the geometry of GPS SV's. The satellite geometry has a larger impact on attitude solution than the measurement errors. There are several Dilution of Precision (DOP) methods to estimate the measurement accuracy from GPS and the solution accuracy.

- **Inter antenna distance**

The accuracy of attitude parameter estimation is also related to the inter-antenna distance, the number of antennas and the geometry of the antenna array. The simplest way to dilute the impact of ranging errors and thus to improve the accuracy of attitude estimation is to increase the inter-antenna distance. Hence, the antenna separation becomes larger the enhancement in pitch estimation is more significant that than in heading. However, the extension of inter-antenna distance has a negative effect on ambiguity resolution. The ambiguity search region expands dramatically with the extension of inter-antenna distance, which inevitably leads to an increased complexity in ambiguity identification as well as to a larger consumption of computational power.

3.2 Doppler shift in GPS measurements

The Doppler shift or effect is the change in change in frequency of a wave for an observer moving relative to its source. Due to this effect, the received frequency of the wave will be relatively higher than the frequency of the wave at its origin. In case of fast moving GPS SV's, there will be a Doppler shift in the frequencies of signals at receivers. This includes both pseudorange and carrier phase measurements [14]. The magnitude of the Doppler effect changes due to earth curvature. The noise bandwidth of the GPS receiver should cope with the Doppler shifts in the receiving GPS signals.

Generally, receivers are designed to adjust the bandwidth according to the received signal. An oscillator in the GPS receiver is used to the track the pseudorange and carrier phase. The drifting of the oscillator is observed in Doppler measurement. The calculations involving Doppler shifts are very complex due to their dependencies. Due to this, the accuracy is very less. For this reason, the Doppler shift is rarely used in attitude determination solutions. The Doppler shift in frequency is given by:

$$D_i = -(\frac{v_i - v_u}{c} \cdot \bar{I}_i^T)\, L_1 \quad (3.2)$$

Where, v_i is the velocity of the GPS SV

v_u is the velocity of the GPS receiver

c is the speed of light

$\bar{I}_i^T$ is the line of sight vector

And, L_1 is the carrier frequency, which is 1.57542*GHz*

The line of sight vector is given by:

$$\bar{I}_i^T = \frac{r_i - \hat{r}_u}{|r_i - \hat{r}_u|} \quad (3.3)$$

Where, r_i is the location of i^{th} GPS SV

And, $\hat{r}_u$ is the estimated location of the GPS receiver.

3.3 Differencing techniques

There are various methods to eliminate common-mode errors. Most common of such methods is differencing of GPS measurements. These methods also yield true integer ambiguities. The errors caused due to transmission and propagation are removed when the measurements are differenced without introducing any unknown terms. There are two types of differential techniques [15]. They are:

- Single differences
- Double differences

Single differences method

The single difference measurement is the difference between the signal produced by the same GPS SV and received by two different GPS receivers. The unknown parameters in measurements would remain same as before, except that relative clock bias between the two receivers will be considered. Since the GPS signals received by two nearby receivers tend to have nearly same propagation paths, their propagation errors will be similar. Hence, the common errors will be removed when the measurements are differenced but without introducing any addition unknowns.

This will result in reduction of satellite clock, orbital and atmospheric errors. Consider that receivers A and B from figure 3.2 are receiving the signal from GPS SV j. Then the single differenced carrier phase measurement is:

$$\Delta\phi_1^{AB} = \phi_1^{A}(t_{RA}) - \phi_1^{B}(t_{RB}) \tag{3.3}$$

Where,

- $\phi_1^{A}(t_{RA}) = \left[\gamma_{1A}^{j}(t_{RA}) - \psi_{1A}^{j}(t_{RA} - \delta t_{RA}) + \text{errors}\right]$
- $\phi_1^{B}(t_{RB}) = \left[\gamma_{1B}^{j}(t_{RB}) - \psi_{1A}^{j}(t_{RB} - \delta t_{RB}) + \text{errors}\right]$

Both the terms will same transmitted carrier phase, initial satellite clock error. Hence, theses errors will be eliminated when differenced.

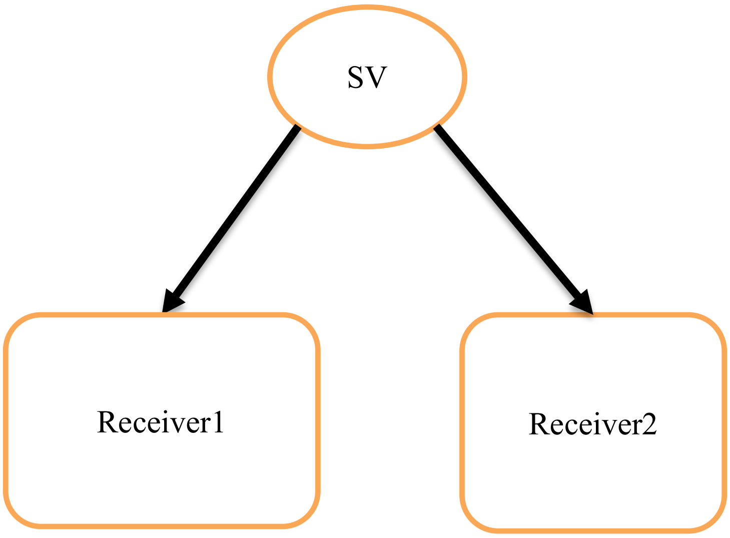

Figure 3.2: Typical single difference scenario

The distance between GPS receivers will affect the error terms in measurement calculations. Less the difference between GPS receivers will reduce the ionosphere errors. If the receivers are linked to each other, then, they can have their sampling clocks to be phase- locked to a common oscillator. This can be used to guarantee that the single –differenced carrier phase ambiguity is an integer. However, there are a few disadvantages to this type of measurement. The actual number of carrier cycles used in measurement of carrier phase cannot be determined.

In case of pseudorange measurements, the single difference measurement is given by:

$$\Delta\rho_1^{AB} = \rho_1^A(\mathrm{t_{RA}}) - \rho_1^B(\mathrm{t_{RB}}) \tag{3.4}$$

Where,

- $\rho_1^A\,(\mathrm{t_{RA}}) = \left[\rho_A^j + \mathrm{c}\,.\,(t_{RA} - \delta t_{RA}) + \text{errors}\right]$
- $\rho_1^B\,(\mathrm{t_{RB}}) = \left[\rho_B^j + \mathrm{c}\,.\,(t_{RB} - \delta t_{RB}) + \text{errors}\right]$

The main advantage of differencing pseudorange measurements is that it reduces the magnitude of errors. Reducing the errors results in better positioning and navigation solution.

Alternatively, single difference measurement can be measured as differences between signals generated by two different GPS SV's that are received by a single receiver. In this case, the two observations have same receiver clock, which negates the clock errors.

Double differences method

Double differences method is a combination of two single differences. Double differences are measured by taking two between-receiver single differences and differencing these between two satellites ie it is measured by subtracting two single differences measured on two satellites (shown in figure 3.3). Double differencing between two receivers can be used to eliminate common receiver generated errors from the single difference between two receivers.

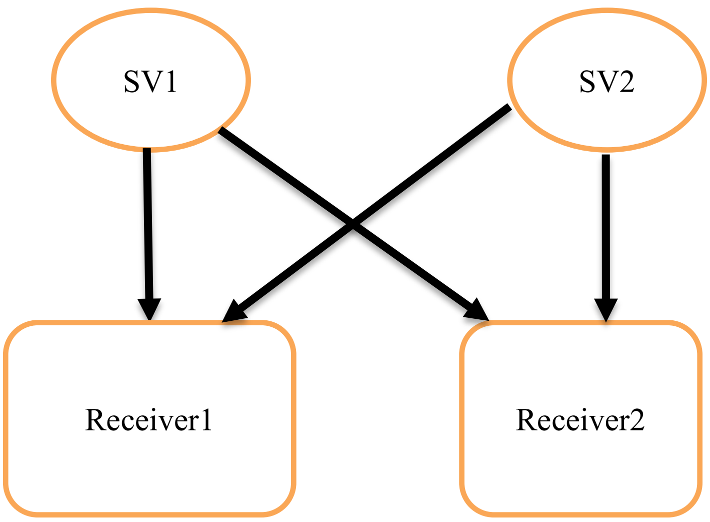

Figure 3.3: Typical double difference scenario

The carrier phase double difference is given by:

$$\nabla\Delta\phi_{AB}^{ij} = \Delta\phi_{AB}^{i} - \Delta\phi_{AB}^{j} \tag{3.4}$$

Where,

- $\Delta\phi_{AB}^{i} = (\Delta\phi_{A}^{i} - \Delta\phi_{B}^{i})$
- $\Delta\phi_{AB}^{j} = (\Delta\phi_{A}^{j} - \Delta\phi_{B}^{j})$

This subtraction allows double-differences to be used even for receivers whose sample times are not synchronized. If the baseline between errors A and B is short, it leads to reduction in ionospheric errors. This implies that the double differenced carrier phase ambiguities can be resolved as integers. Other error sources such as multipath errors are too small to result in integer ambiguity.

The pseudorange double difference is given by:

$$\nabla\Delta\rho_{AB}^{ij} = \Delta\rho_{AB}^{i} - \Delta\rho_{AB}^{j} \tag{3.5}$$

Where,

- $\Delta\rho_{AB}^{i} = (\Delta\rho_{A}^{i} - \Delta\rho_{B}^{i})$
- $\Delta\rho_{AB}^{j} = (\Delta\rho_{A}^{j} - \Delta\rho_{B}^{j})$

The process of double differencing can be repeated for more pairs of satellites resulting in three double differenced measurements, which can be used to solve the difference between GPS SV and receiver locations. Double differencing is a relative positioning technique. Hence, to determine the absolute position of GPS SV, it is important to know the position of GPS receiver.

Both single and double difference methods are valid tools position determination. In both these methods, it is extremely important that data from two must be collected from two receivers simultaneously [10][15].

Chapter 4

Various integer ambiguity resolution methods and baseline estimation

4.1 Integer ambiguity in GPS position estimation:

The accuracy of attitude solution depends on the carrier phase of the GPS satellite signal. Signal blockage and noise disturbance that occur during vehicle movement may cause loss-of-lock of carrier phase. This loss of signal leads to integer ambiguity. Integer ambiguity refers to the unknown number of wavelengths of carrier signal in a set of GPS measurements received by a single receiver. This is a common recurring problem. During attitude determination using a global positioning system (GPS), cycle slips occur due to the loss of lock and noise disturbance. Integer ambiguity resolution is the process of resolving unknown cycle ambiguities to integers in GPS applications. This process is an important and crucial step in attitude determination, which results in cm-level precision [16].

If the integer ambiguities are not resolved and if one or more cycle slips occur, accuracy of the attitude solution is affected. There are two different methods to estimate the carrier phase ambiguities for inter-antenna vectors:

- Motion based ambiguity resolution [5]
- Vector (baseline) based ambiguity resolution

Vector based ambiguity resolution (also known as baseline estimation) is most commonly used method and is discussed in this thesis. This type of ambiguity resolution method is independent of platform. This was initially developed for differential carrier phase positioning and can be used for instantaneous ambiguity resolution. This was introduced by Hatch in 1991 [19].

Cohen introduced a traditional motion-based technique of integer ambiguity resolution [5]. According to the algorithm, "quasi-static" integer resolution, developed by Cohen [5], either GPS line of sight motion or vehicle motion attribute to the changes in differential carrier phase measurements evenly. The motion-based ambiguity resolution method improves the precision and reliability, when compared to the vector based ambiguity resolution. The disadvantage of this method lies in its high computation time, which, is not desired.

There are three important steps for any ambiguity resolution for attitude determination in GPS systems:

- Defining the search region for ambiguities
- Forming the ambiguity combinations
- Selecting the correct ambiguity set.

Since the GPS antennas are closely spaced and their locations are fixed, the geometry of the antenna array can be determined using any conventional surveying methods. In a GPS attitude determination system, the multiple GPS antennas are closely spaced and the locations of the antennas are fixed with respect to each other during motion. This array information is used as existing information to determine the carrier phase integer cycles in ambiguity resolution. Any ambiguity search region should include an ambiguity combination and have a small volume that will minimize the computation effort. In general, for GPS attitude determination application, search volumes such as sphere, cube or an ellipsoid are used. Due to lack of attitude information in GPS systems, a sphere search region is most commonly used. The next step is to determine all the ambiguity sets that fall in the search zone [17].

The integer ambiguity resolution process should have the following properties:

- The ambiguity resolution process should have maximum computation efficiency. Under normal circumstances, an integer ambiguity can be identified and corrected in a single measurement epoch.
- Only the correct ambiguity combination should be sorted out. If any wrong ambiguities have been selected, some exception handling processes should be incorporated to detect, identify and correct the ambiguities.

To verify the ambiguity result and the consistency of the estimated vector angles to, a minimum of two antenna vectors should be determined. If the inter-antenna distance and its radius are specified, then the location of secondary antennas should fall on the shell of the sphere search region. Another type of test exists that utilizes the statistical properties of the ambiguity sets to detect the true combinations. It is important to note that integer ambiguities are not a function of time. They remain constant as long as there are no cycle slips. Various tests are used in this phase to make sure only the correct ambiguity set is determined. These tests are based on antenna array constraints and the statistical properties of the true ambiguity set. The estimated inter-antenna distances from the true ambiguity set should be consistent with the existing values within a tolerance band to cover carrier phase measurement errors.

There are various methods to minimize ambiguity. All of these methods are based on least squares estimation method. Few of them discussed in this research are:

- Least Squares Ambiguity Search Technique
- LAMBDA method

- MC-LAMBDA method

4.2 Least Squares Ambiguity Search Technique

Least squares estimation uses integer approximation of conditional least-squares technique. This minimizes search combinations and computation speed, making the ambiguity estimation problem easier to solve. This method assures improvement in precision in comparison with the original integer ambiguities. Initially, a set of all ambiguity combinations is formed, using primary double differences measurements at a certain epoch. Then, by using the primary observations, the false combination is eliminated after comparing the estimated antenna vector length with the known value. For each primary ambiguity observations, a secondary ambiguity set is formed. This division into primary and secondary measurements reduces the search space and helps in validating the phase ambiguities[18].

Since the antenna vector baseline value is known, the double difference ambiguities are directly computed. The criterion to select the four primary satellites depends on the satellite distribution and the corresponding carrier phase measurement quantities. Selecting four primary satellites is required for the division of ambiguity sets into primary and secondary. The ambiguities for the secondary set are solved using the solution obtained by solving the primary ambiguity set.

This method was introduced by Hatch [19]. According to the originally proposed idea, the sum of squares of double difference carrier phase residuals should be minimum for correct ambiguity set. There are three steps involved in this process. They are:

- Estimate the unknown point coordinates

- Define the ambiguity sets
- Apply the ambiguity search algorithm
- Discriminate and test the validity

4.2.1 Algorithm for Least Squares Ambiguity Search Technique (LSAST)

Let us consider the following equation from three primary double difference group [19]:

$$M_p = B_p * X_p \tag{4.1}$$

Where,

- M_p is the measurement vector of the primary set
- B_p is the direction cosines matrix of the primary set
- X_p is the solution vector of the primary set

Equation (4.1) can also be re-written in the following form:

$$\begin{bmatrix} \emptyset^1 + N^1 \\ \emptyset^2 + N^2 \\ \emptyset^3 + N^3 \end{bmatrix} = \begin{bmatrix} c_i^1 & c_j^1 & c_k^1 \\ c_i^2 & c_j^2 & c_k^2 \\ c_i^3 & c_j^3 & c_k^3 \end{bmatrix} * \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} \tag{4.2}$$

Where,

- N is the ambiguity
- Ø is the phase double difference
- C is the direction of cosines to the satellites
- $\delta x, \delta y, \delta z$ are the direction of estimated baseline correction
- Superscripts 1,2 and 3 are the satellites
- Subscripts *i,j* and *k* are the x,y,z directions

The solution vector of the primary set can be derived from the above equation (4.1) and (4.2) is given as:

$$X_p = B_p^{-1} * M_p \tag{4.3}$$

Due to different combinations of N(ambiguity), for different measurement vectors, M_p, the value of B_p^{-1} remains constant. Hence, the general measurement vector can be given as:

$$M_p^T = [\alpha \;\; \beta \;\; \gamma] \tag{4.4}$$

And, the final solution to the measurement vector is given by,

$$X_p = \alpha X_1 + \beta X_2 + \gamma X_3 \tag{4.5}$$

Next, a secondary group is formed to eliminate the incorrect combinations. This is done by varying the values of α ,β and γ in loops. Let Y_s be the innovation vector for the secondary group. Subscript, s , represents the secondary group. Then,

$$Y_S = M_S - B_S X_S \tag{4.6}$$

The residual vector, R, is given by,

$$R = Y_c - B_c * \Delta X \tag{4.7}$$

The estimated variance for measuring the quality of potential solutions is given by:

$$q = \frac{R^T R}{m-3} \tag{4.8}$$

Where, m is the total number of double differences. All the solutions that are less than the value of q are eliminated.

4.3 LAMBDA method

This method is known as Least squares Ambiguity Decor-relation Method. This method involves decor-relation of integers using mathematical transformations for estimation. This method is an extension of the general least- squares estimation of integer ambiguities. It divides the states of ambiguities and covariance matrix, which makes this method more efficient. Teunissen introduces this method in 1996 [17]. This method is considered very efficient for the following advantages it offers [20]:

- Transforms the states of ambiguity to maintain minimum correlation.
- Reduction of number of possible ambiguity combinations
- High probability of finding the correct integer values for ambiguities.
- Reduction in search space of integer values.

LAMBDA is a three-step process [20]:

- Compute the float solution to discard the integer nature of ambiguities
- Determine the integer values for the ambiguity sets
- Compute the final solution, using the values obtained from this method.

By maintaining minimum correlation between the states of ambiguity, number of ambiguities reduces which in turn, improves the efficiency of this method by reducing the computation time. Also, the number of possible ambiguity combinations is reduced in one epoch. Generally, the search area used is an ellipsoid. The ellipsoid is gradually transformed into a sphere. Consider the following linear equation for a double difference model with short baselines [20][21]:

$$y = Aa+Bb+ \varepsilon \tag{4.9}$$

Where,

- A and B are design matrix for ambiguity and baseline matrix respectively
- a is the integer ambiguity double difference vector
- b is the baseline increment vector
- y is observed minus computed double difference.
- ε is the un-modeled errors vector.

From equation (4.9), the double difference observation vector y, can be re-written as:

$$[\mathrm{B} \quad A] \begin{bmatrix} \mathrm{b} \\ \mathrm{a} \end{bmatrix} = \mathrm{y} + \varepsilon$$

In order to fix the ambiguities in a i.e. to find the integer solution to a, it is necessary to find a solution for $\begin{bmatrix} b \\ a \end{bmatrix}$ matrix [22].

Let us consider two parts to

- $\hat{a}$, the real values of the vector a
- $\breve{a}$, the integer values of the vector a
- a_r , the remainder values of vector a

$$\begin{bmatrix} \hat{b} \\ \hat{a} \end{bmatrix} = \begin{bmatrix} B^T B & B^T A \\ A^T B & A^T A \end{bmatrix}^{-1} \begin{bmatrix} B^T \\ A^T \end{bmatrix} y \qquad (4.10)$$

Where ,

- $Q_{\hat{b}} \ = \ B^T B$
- $Q_{\widehat{ba}} \ = B^T A$
- $Q_{\widehat{ba}}^T \ = A^T B$
- $Q_{\hat{a}} \ = \ A^T A$

The condition to find the solution for vector, $\breve{a}$ is:

$$(\hat{a} - a)^{-1} Q_{\hat{a}}^{-1} (\hat{a} - a) = \text{ Minimum over integer vector a}$$

and , $$(\hat{a} - a)^{-1} Q_{\hat{a}}^{-1} (\hat{a} - a) < \chi^2 \qquad (4.11)$$

where, χ is the search volume

The solution to the equation (4.11) will be both $\hat{a}$, the real values of the vector a , and the covariance matrix , $Q_{\hat{a}}$.

4.3.1 Algorithm for LAMBDA method

Step 1: Decomposition of the covariance matrix, $Q_{\hat{a}}$

Let $Q_{\hat{a}} = LDL^T$ (4.12)

Step 2: Compute the initial size of the search area as squared distance of float vector, partially rounded to the float vector in the covariance matrix, $Q_{\hat{a}}$

Step 3: Apply integer Gauss transformations and permutations to form a matrix, Z, such that, $Q_{\hat{a}} = Z^T Q_{\hat{a}} Z$ (This is known also known as decor-relation)

Step 3: Compute the transformed and shifted ambiguities. The shifted ambiguity vector will be, $a_s = Z^T a_r$

Step 4: The transformed and decor-related matrix $Q_{\hat{a}}$ is calculated using a_s

Step 5: The final result of the integer search is $\bar{a} = (Z^{-1})^T \breve{a}$

The ambiguity vector must be a column vector with n ambiguities. Then, the covariance matrix must always be a square matrix of order n x n. The ambiguity number n is equal to the number of satellites minus one, multiplied by the used frequencies values, and the number of baseline components b is three, in case of a static receiver, or a multiple of three, in case of a moving one [22].

4.4 Multivariate Constrained-LAMBDA Method

This method is an extension of LAMBDA method, also proposed by Teunissen [21]. It is known as Multivariate constrained – LAMBDA method. In this method, both baseline lengths and relative orientations between the antennas are considered as geometrical constraints. It can be applied to any number of linear and non-linear integer ambiguity sets. This method is also uses single frequency, single epoch [23].

The minimization problem is represented by:

$$Z^c = \arg\min C(Z) \tag{4.13}$$

Where,

- Z is the integer matrix, $Z = Z^{nxm}$
- C(Z) is cost function

The cost function C(Z) is given by :

$$C(Z) = ||\min (z - \hat{z})||^2_{Q_{\hat{z}}} + ||\text{vec}\,(\hat{R}(Z)) - \text{vec}(\check{R}(Z))||^2_{Q_{\hat{R}}} \quad (4.14)$$

Because of the minimization of cost function, it makes the search strategy inefficient, due to limitation of search space. This increases the computation time. This avoids the limitation; two search strategies called " Search and shrink approach" and "Expansion approach" are used. Both these strategies adjust the size of the search space by shrinking and expanding the integer ambiguities sets. This allows a faster and efficient search. The method solves for both the integer ambiguities and the orientation angles. This maximizes the efficiency of integer ambiguity resolution.

4.5 Advantages and disadvantages of LSAST and LAMBDA methods

The ambiguity resolution methods, LSAST and LAMBDA have advantages and disadvantages, which must be considered during calculations. Table 4.1 shows the various advantages and disadvantages of LSAST and lambda methods.

Method	**Advantage**	**Disadvantages**
Linear Squares Ambiguity Search Technique	• Only combinations of primary satellites are recorded, and others are rejected. • Minimized computation speed. (Decreasing the number of combinations of integer ambiguities)	• In case of large error, estimated measurement and actual measurement have a huge difference. • This method can be applied to only simple difference measurements, but not to DGPS [24].

LAMBDA	• Mathematical integer transformation minimizes the correlations between ambiguities. • Number of ambiguity sets is reduced.	• Calculation of standard deviation in least squares estimation cannot be applied for fractional ambiguities. • Large number of epochs to resolve ambiguities.

Table 4.1:Advantages and Disadvantages of LSAST and LAMBDA methods [18]

4.6 Baseline component estimation

Baseline vectors are relative position vectors that are useful in attitude determination. Once all the existing ambiguities have been resolved, baseline components can be estimated. In this section, two different baseline component estimation methods are highlighted:

4.6.1 Baseline estimation using least-squares estimation

Steps involved in baseline estimation using this method are [22]:

Step 1: Estimate the primary baseline components using the model described in chapter 2

Step 2: Compute the double difference observations

Step 3: Estimate the float ambiguities using LAMBDA

Step 4: Compute the baseline components iteratively for required number of epochs using the formula:

$$b = (H^T R^{-1} H)^{-1} H^T R^{-1} [\nabla\Delta\varphi - \lambda \cdot \nabla\Delta N]$$

Where,

- b is the baseline vector either in XYZ or ENU or NED coordinate frames
- H is the design matrix built by stacking the transpose of the line of sight vector to the i^{th} satellite
- R is the noise covariance matrix
- $\nabla\Delta N$ is the double difference integer ambiguity vector
- λ is the wavelength on L1(.19029367 m)

Step 5: Transform the baseline coordinates into desired coordinate frames.

4.6.2 Baseline estimation using extended Kalman filter [22][25][26]

The main purpose of a filter is to compute the raw DGPS data into a baseline vector.

The general state equations for Kalman filter are given by:

$$X_k = \varnothing_{k-1} \cdot X_{k-1} + W_k$$

$$Z_k = H_k \cdot X_k + V_k$$

Where,

- z_k is the measurements vector at time k
- H_k is the design matrix at time k
- x_k is state vector at time k
- v_k is measurement noise with covariance R
- φ_k is the transition matrix
- w_k is the process noise with covariance Q

Steps involved in baseline estimation using this method are:

Step 1: Prediction of initial state vector and covariance matrix using equations

$$X_k = \O_{k-1} \cdot X_{k-1}$$

$$P_k = \O_{k-1} . P_{k-1}$$

Step 2: Update the Kalman gain, K

$$K = P_k * H ' * inv(H * P_k * H^T + R)$$

Step 3: The next state vector and covariance matrix are updated as outputs

$$X_o = X_0 + K*(Z - X_k)$$

$$P = [I - K*H]*P$$

This process is repeated recursively for more precise results.

The state vector, X contains the estimated baseline components and is defined as:

$$X = [\ x, y, z, b, d]$$

Where,

- x , y, z are the coordinates of the baseline estimates
- b is the clock bias
- d is the clock drift

Chapter 5

Results, Summary and Future work

5.1 Results

In the previous chapters, different methods of position estimation, different integer ambiguity fixing methods were described. In this chapter of thesis, results from the below mentioned are plotted:

- Receiver position estimation
- Baseline estimation using Least Square Estimation (LAMBDA ambiguity resolution)
- Baseline estimation using Kalman filter (LAMBDA ambiguity resolution)
- Different baseline estimations from both the methods described above
- Attitude components (yaw, pitch and roll) estimation
- Comparison of attitude components estimated using different integer ambiguity methods

5.2 Receiver position estimation using pseudorange measurement

In this section, the coordinates (X, Y, Z) of the receiver location from the observed pseudoranges are estimated. All the coordinates are in earth fixed coordinate system (EFCS). The variation of the receiver position relative to epochs (twenty-two epochs) is shown in figure 5.1. Each receiver position is computed from the satellite positions iteratively over twenty-two epochs using least squares principle. In figure 5.2, variations in baseline components are plotted using P2 pseudoranges from the receivers. Table 5.1 shows the mean receiver position over twenty-two epochs. Table 5.2 shows the computed values of baseline components over twenty-two epochs.

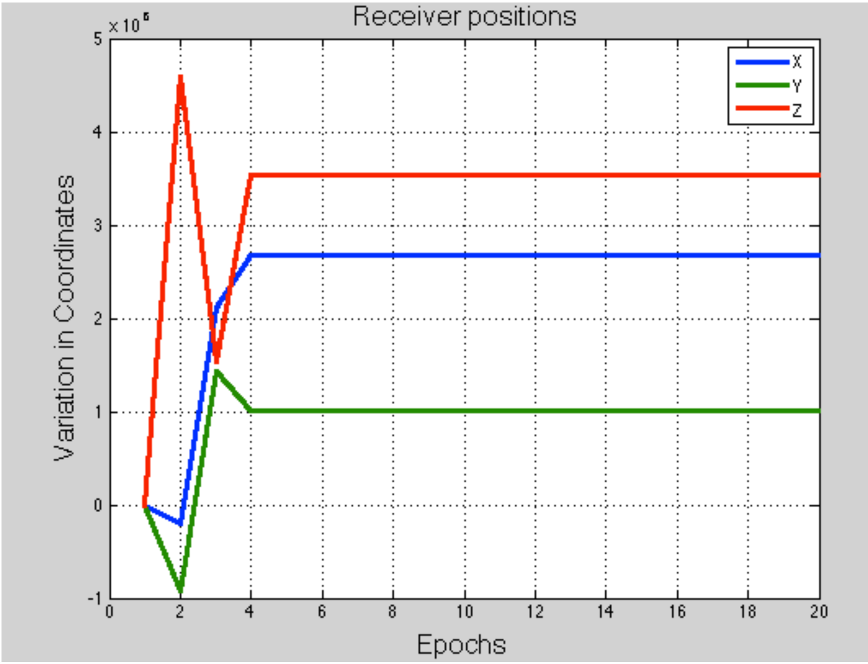

Figure 5.1: Variation in receiver positions using pseudoranges

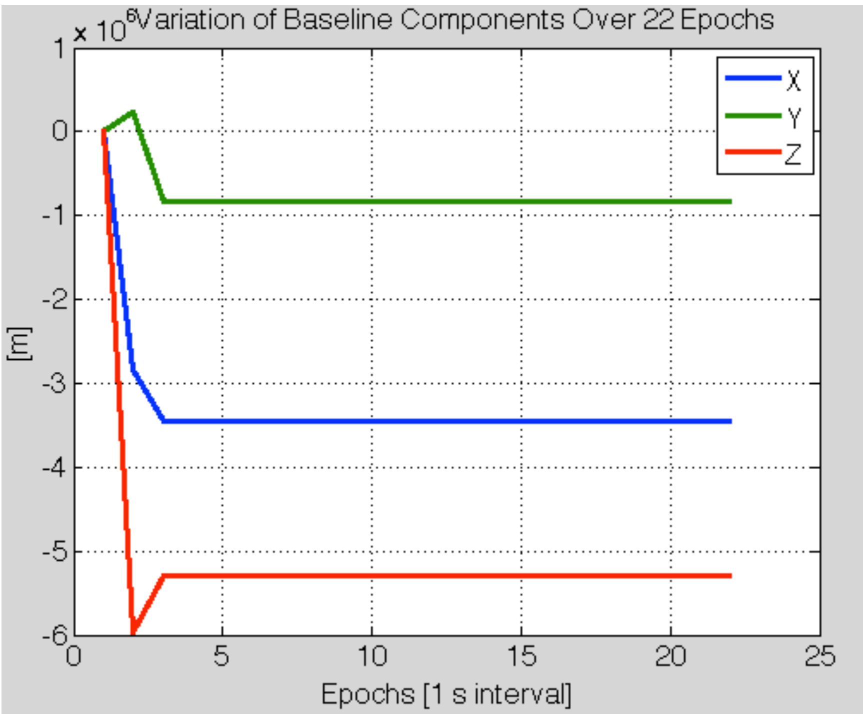

Figure 5.2: Variation in baseline components using pseudoranges

X	Y	Z
3123219.393	477069.771	5103745.721

Table 5.1: Mean receiver position over 22-epochs

X	Y	Z
0.571 mm	-7.724 mm	0.622 mm

Table 5.2: Baseline Components as Computed From 22 Epochs

5.3 Estimation of baseline components using Least Square Estimation (LAMBDA ambiguity resolution)

In the section 5.1, the receiver position and baseline components were using pseudorange measurements (P2). However, in estimating baseline components using carrier phase measurements (L1 or L2), ambiguities exist. First, the ambiguities need to be resolved. In this section, the ambiguities are resolved using LAMBDA using the steps mentioned in 4.3.1, and the baseline components are estimated using recursive least squares estimation technique. The algorithm described in the section 4.6.1 for baseline estimation using linear squares estimation method is applied.

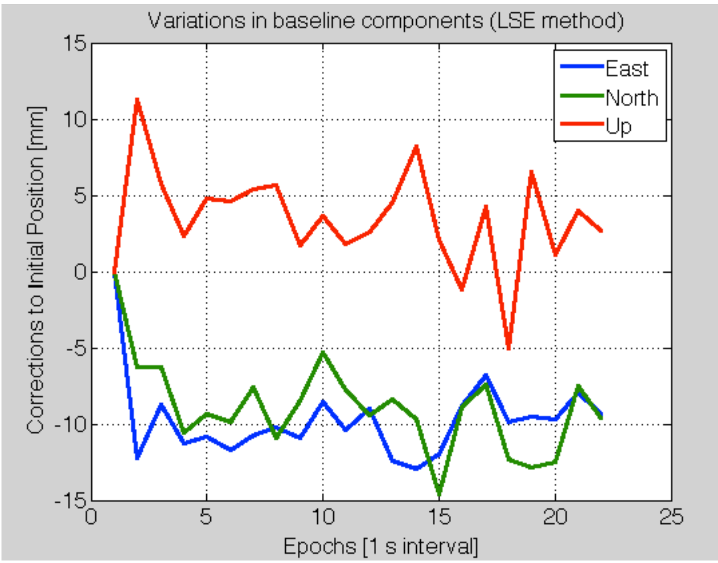

Figure 5.3: Variation in baseline components using carrier phase (LSE) [9]

It can be observed that the resulting variations in baseline components are improved by a factor of 1000. Figure 5.3 shows the variations in baseline components using carrier phase measurements for 22 epochs and least square integer ambiguity estimation.

5.4 Estimation of baseline components using Extended Kalman Filter (LAMBDA) ambiguity resolution

In this section of thesis, baseline components are estimated using an extended Kalman filter in LAMBDA for integer ambiguity resolution. Figure 5.3 shows the estimated variation in state vectors with variation in epochs. The number of epochs used in this plot is 22. The algorithm described in the section 4.5 for baseline estimation using extended Kalman filter estimation method is applied.

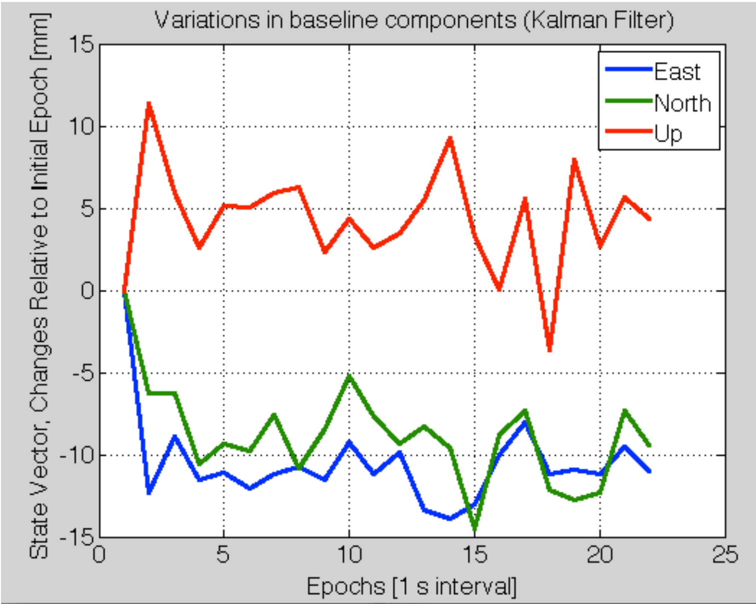

Figure 5.4: Variation in baseline components using carrier phase measurements (Kalman filter)[9]

5.5 Different baselines from Least Square and Kalman filter in LAMBDA

Table 5.3 highlights the different the estimated baseline components by using carrier phase measurements at the end of 22nd epoch (least squares and Kalman filter ambiguity resolution methods).

Estimated baselines	carrier phase (LSE)	carrier phase (KF)
X	1.1404 mm	2.218 mm
Y	-8.303 mm	-7.893 mm
Z	0.704 mm	0.346 mm

Table 5.3: Baseline components carrier phases

5.6 Estimation of attitude components

Once the body frame baselines are estimated, the last step will be actual attitude determination. It means the computation of the attitude angles i.e. yaw, pitch and roll. The accuracy of the attitude solution is directly proportional to position accuracy. For attitude determination, initial guess is required. Hence, the coordinates the antennas are first calculate and then, the attitude parameters. The relationship between baseline estimates and attitude parameters is defined as:

$$b_i = \begin{bmatrix} c_r c_y - s_r s_p s_y & c_r s_y + s_r s_p c_y & -s_r c_p \\ -c_p s_y & c_p c_y & s_y \\ s_r c_y + c_r s_p s_y & s_r s_y - c_r s_p c_y & c_r c_p \end{bmatrix} \tag{5.1}$$

Where,

- Operators c and s represent cos () and sin (),
- The subscripts y, r and p represent yaw, roll and pitch, respectively
- b_i denotes the position of antenna

Figure 5.4 shows the estimation of attitude parameters by using the baseline components obtained using carrier phase measurements and ambiguity resolution by least squares estimation in LAMBDA. The x-axis shows the epochs (in seconds) and the y-axis shows the estimated Euler angles (in the units of degree).

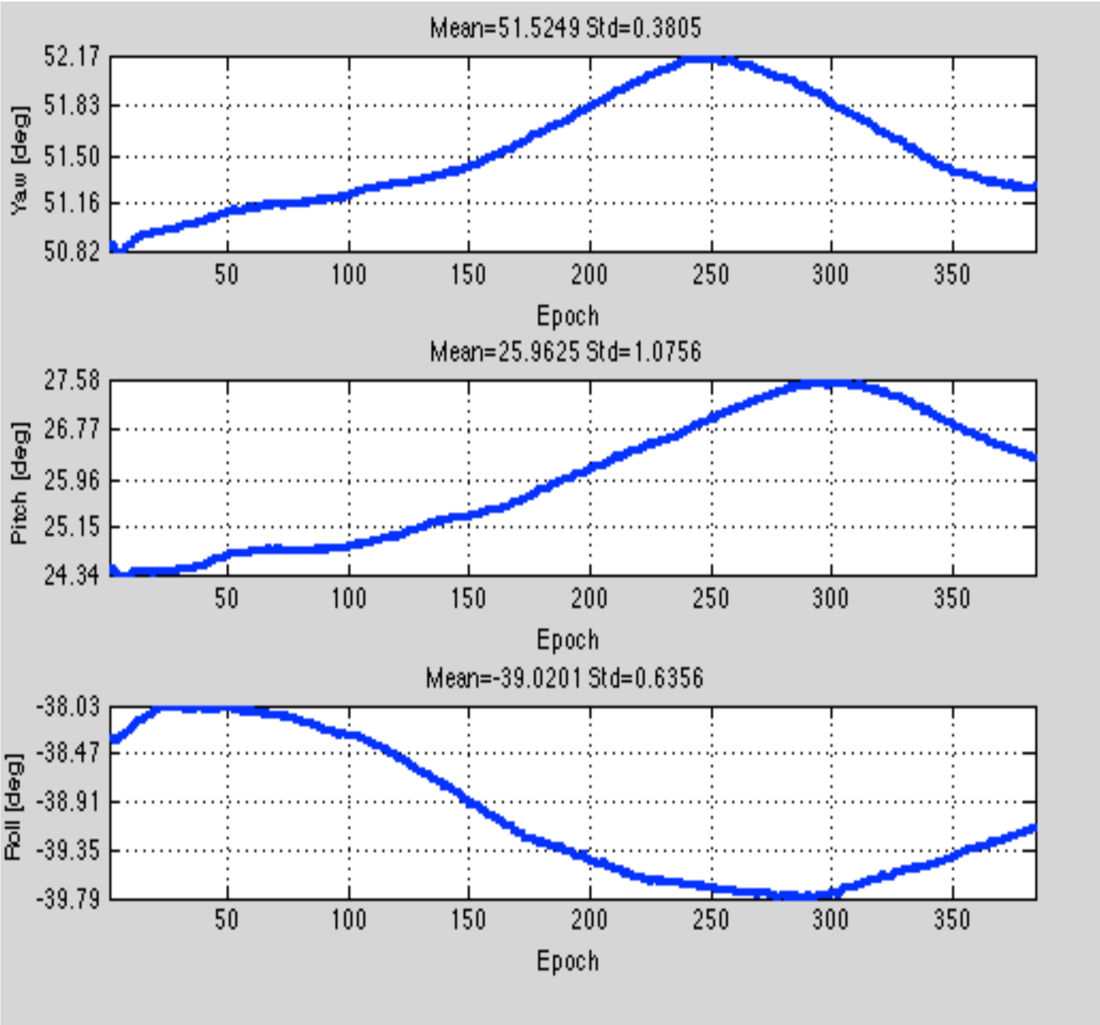

Figure 5.5: Attitude parameters using LAMBDA (Least squares integer ambiguity resolution) [27]

Figure 5.5 shows the estimation of attitude parameters by using the baseline components obtained using carrier phase measurements and ambiguity resolution by Kalman filter in LAMBDA. The x-axis shows the epochs and the y-axis shows the estimated Euler angles in the units of degree.

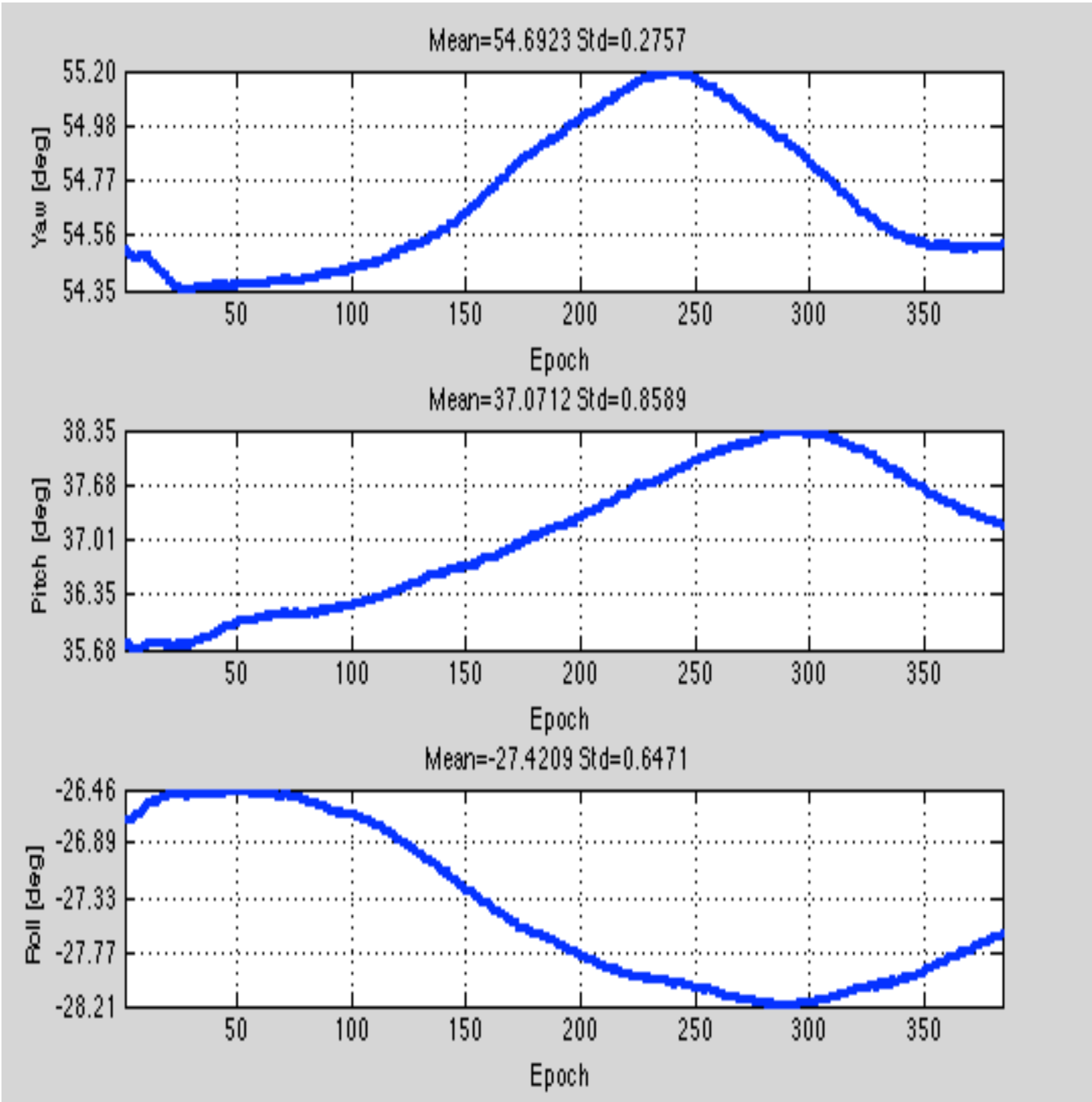

Figure 5.6: Attitude parameters using LAMBDA (Kalman Filter ambiguity resolution) [27]

5.7 Different attitude components obtained from Least Square and Kalman filter in LAMBDA

In the previous sections, a variation in attitude components with variations in epochs (around 400 epochs) was demonstrated. The mean and standard deviation of the attitude components, yaw, pitch and roll estimated using both Least Squares and Kalman Filter estimation techniques are shown in Table 5.4.

Method	Yaw	Pitch	Roll
LAMBDA(LSE)	mean=51.5249 std=0.3805	mean=25.9265 std=1.0756	mean=39.0201 std=0.6356
LAMBDA(K.F)	mean=54.6923 std=0.2757	mean=37.0712 std=0.8589	mean=27.4209 std=0.6471

Table 5.4: Estimated attitude parameters using LAMBDA

It can be observed that the standard deviation of the three components is less than one. This is due to the presence of noise.

5.8 Summary

In this thesis, attitude determination using differential GPS system is discussed. Differential GPS and navigation system using carrier phase measurements is widely used and one of the most accurate ways to estimate position and attitude of any object in space (satellite, aircraft etc.). Using this technique, the visibility of satellites and integrity of signals are preserved. The topics presented in this thesis focus largely on methods that provide accurate and highly precise solutions. This thesis has implemented the below mentioned major aspects involved in the attitude determination process using GPS systems:

- Different GPS measurements and processing of GPS measurements
- Mathematical modeling of GPS measurement and the GPS systems
- Error analysis in position and GPS measurements
- Integer ambiguity resolution techniques
- Baseline estimation using least squares estimation and Kalman filter
- Final attitude determination

It has been demonstrated that the implemented algorithms are capable of calculating position and attitude for differential GPS system. Rapid ambiguity is the key to precise GPS position estimation. In chapter 4, two methods for ambiguity resolution namely LSAST and LAMBDA are presented. The LAMBDA method has been implemented and from the results, it is evident that it is an efficient approach for ambiguity resolution. The LSAST method is least preferred due to limitations like search space, increase in errors due to phase variations, etc. The LAMBDA method is also capable of monitoring and detecting cycle slips and fixing them. The integer ambiguity resolution technique presented in this thesis is examined in the context of real-time applications.

Chapter 4 also presents different baseline estimation techniques. They provide an adaptive approach to navigation that is suitable for both stand-alone and real-time applications. Matlab has been extensively used in achieving the results.

5.9 Future work

More research needs to be done to improve the performance of GPS-based attitude determination systems. The following are recommendations for future work in this area:

- The applied procedure in this thesis can be extended to four antennas and the results and performance can be studied.
- The accuracy and reliability of GPS-based attitude systems can be significantly improved if the carrier phase multipath errors in kinematic conditions can be investigated.
- The advantages and disadvantages of integer ambiguity resolution using MC-LAMBDA should be studied and applied in real time applications.

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